CHAPTER: ARITHMETIC SEQUENCE

Learning outcomes:

Students will be able to:

• recognise arithmetic sequences in a variety of contexts

• recognise sequences that are not arithmetic

• apply their knowledge of arithmetic sequences in a variety of contexts

• apply the relevant formula in both theoretical and practical contexts

• calculate the value of the first term (a), the common difference (d) and the general term (Tn) of an arithmetic sequence from information given about the sequence

ASSERTION REASON:

DIRECTION: In the following questions a statement of **ASSERTION (A)** is followed by a statement of **REASON (R).** Choose the correct answers out of the following choices.

a. Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true and the Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.

Q1. ASSERTION (A): In an AP, with a=17,d= -5 then the 6^{th} term will be -8.

REASON(R) : The a-d, a, a+d are the three numbers of an AP.

Q2. ASSERTION (A): The 11^{th} term of an AP is 48 whose first term is 8 and common difference is 4.

REASON(R): The n^{th} term of an AP is a_n=a + (n-1)d where a is the first term and d is the common difference.

Q3. ASSERTION (A): The 15^{th} term of an AP is 40, 37, 34,….. is the first negative term of AP.

REASON(R) : The n^{th} term of an AP is a, a + d, a + 2d…….. is given by

$S_n=\frac{n}{2}\{2a+(n=-1)d\}$.

Q4. ASSERTION (A): -7, $-7/2$,0,$7/2$ is in AP

REASON(R): The terms of an AP cannot have both positive and negative rational numbers.

Q5. Assertion (A): If S_n is the sum of first n terms of an AP, then its n^{th} term a_n is given by:

$$\boldsymbol{a_n = S_n - S_{n-1}}$$

Reason(R): The 10th term of the AP: 5,8,11,14,.... Is 35

Q6. Assertion (A): Sum of first 10 even natural numbers is 120.

Reason(R): If **a** is the first term, 1 is the last term and **d** is the common difference of an AP then n^{th} from the end is given by $1 - (n-1)d$

Q7. **Assertion:** Let the positive numbers p, q, r be in A.P., then 1/qr, 1/pr, 1/pq are also in A.P.
Reason : If each term of an A.P. is divided by pqr, then the resulting sequence is also in A.P.

Q8. Assertion: Sum of n terms of an Arithmetic Progressions is $S_n = 2n - n^2$. Its 10th term is -17.

Reason: Sum of n terms of an Arithmetic Progressions is $S_n = n/2[2a + (n - 1)d]$

Q9. Assertion: 1/7, -2/7, -5/7, -8/7,... forms an Arithmetic Progression.

Reason: If $d_1 = d_2 = d_3$=....., then the given terms form an arithmetic progression.

Q10. Assertion: If the fourth of an A.P., is 21 and the sixth term is 29, then the tenth term is 45.

Reason: If the common difference of an A.P, is 5, then, $a_{18} - a_{13} = 35$.

MULTIPLE CHOICE QUESTIONS

Q11. 37th term of the A.P: 11, 8, 5, …..

(a) 97 (b) −97 (c) 119 (d) −119

Q12. The common difference of the AP $\sqrt{8}$, $\sqrt{18}$, $\sqrt{32}$, ...

a) 10 b) $\sqrt{2}$ c) $\sqrt{3}$ d) $\sqrt{8}$

Q13. Sumit receives ₹ 60 as pocket money for the first week and ₹ 3 more each week than the preceding week. How much does he earns by the 20th week?

a) ₹177 b) ₹1770 c) ₹1170 d) ₹1790

CASE STUDY

Q14. Pooja is celebrating her birthday. She invited her friends. She bought 120 pastries. She arranged the pastries such that in the first row, there are 3 pastries, in the second row there are 5 pastries and in the third row 7 parties and so on.

On the basis of above information, answer the following questions:

1. Find the total number of rows of pastries.
2. Find the difference in the number of pastries in the 7th and 3rd row.
3. How many pastries are there in the 5th row?

Q15. In a class test the teacher asks every student to write an example of A.P. Two students Radha and Gauri write their progressions as –3, –1, 1, 3,…… and 151, 143, 139, …. respectively. Now, the teacher asks various students of the class the following questions on these two progressions. Find the answers to the questions:

(a). Find the 28^{th} th term of the progression written by Radha.

(b). Find the sum of common differences of the two progressions.

(c). Find the 17^{th} term of the progression written by Gauri.

OR

Find the sum of the first 10 terms of the progression written by Radha.

Q16. Arvind wants to buy a two-wheeler and plans to take loan from a bank for this. He repays his total loan of Rs 1,18,000 by paying every month starting with the first instalment of Rs 1000 and increases the instalment by Rs 100 every month. Now answer the following:

a)Find the amount paid by him in 30th instalment is

b) Find the total amount paid by him in the 30 instalments?

c) What amount does he still have to pay after 30th instalment?

d) If total instalments are 40 then amount paid in the last instalment?

e). Find the ratio of the 1st instalment to the last instalment.

Q17. A teacher of a school, told her students about the importance and advantages of walking. Sailu and Arvind decided to go for a walk everyday. Sailu walks for 2km on the first day and increases the distance by 0.2km subsequently whereas Arvind walked 2.5km on the first day increased the distance by 0.3 km subsequently.

(i) What is the distance covered by Sailu on the 10^{th} day?

(ii) What is the total distance covered by Arvind at the end of 30 days?

(iii) Who walked more: Sailu or Arvind at the end of 30 days.

QUESTIONS

Q18. An equilateral triangle is formed by arranging the balls in rows. The first row consists of one ball, the second row consists of two balls and so one. If 669 more balls are added, then all the balls can be arranged in the shape of a square and each of its sides contains 8 balls less than each side of the triangle. Find the initial number of balls.

Q19. Find the sum of all natural numbers amongst first one thousand numbers which are neither divisible by 2 or by 5

Q20. If 8th term of an Arithmetic Progression is zero, then prove that the difference between 28th term and twice of 18th term is zero.

Q21. Sum of 10 terms of an Arithmetic Progression is 205. The ratio of 2nd to 7th term is 2:5. Find the Arithmetic Progressions.

ANSWER KEY:

1. b.
2. b.
3. b.
4. d.
5. c
6. d
7. a
8. b
9. a
10. c
11. b
12. b
13. b
14. 1. The AP is 3,5,7,.....

a = 3, d = 2

Let there are n rows, $S_n = \frac{n}{2}[a + l]$

$$120 = \frac{n}{2}[3 + 2n + 1]$$

$$n^2 + 2n - 120 = 0$$

$$n = 10$$

2. 7th row = $3 + (7 - 1)2$

$= 3 + 12 = 15$

3rd row = $3 + (3 - 1)2$

$= 3 + 4 = 7$

Difference = 15 - 7 = 8

3. Number of pastries in 5th row = $3 + (5 - 1)2 = 3 + 8 = 11$

15. (a) $a = -3$, $d = 2$

$a_{28} = a + 27d$ implies $-3 + 27 \times (2) = 51$

implies $a = 9$ and $d = 4$, Therefore $a_{10} = 45$

(b) Radha's Common difference is 2 and Gauri's Common difference is −4

Therefore, the product of Their common difference is 2× (−4) = −8.

(c) $a = 151$, $d = -4$

$a_{17} = a + 16d$ implies $151 + 16 \times (-4) = 151 - 64 = 87$.

16. OR

$a_{10} = {}^{10}/_{2}[\, 2 \times (-3) + 9 \times 2] = 5[-6 + 18] = 60$

16. a) ₹ 3900 b) 73500 c) 44500 d) 4900 e) 10:49

Q17. Sailu: 2, 2.2,, 2.4, 2.6,…

Arvind: 2.6, 2.8, 3.1, 3.4,…

(i) Distance covered by Sailu on the 10th day = a 10 = a + 9d
= 2 + 9(0.2)
=3.8km

(ii) Total distance covered by Arvind in 30 days = Sn = n/2[2a + (n – 1)d]
S_{30} = 30/2[2(2.5) + 29 x 0.3]
=205.5km

(iii) Distance covered by Sailu in 30 days
S_{30} = 30/2[2(2) + 29(0.2)]
= 147km
Arvind covered more distance.

Q18. Let each side of the triangles has n balls.

Number of balls in triangle = 1+2+3+ ….. $\frac{n(n+1)}{2}$

Number of balls in each side of square = n-8

Total number of balls in square = $(n - 8)^2$

By the given condition,

$$\frac{n(n+1)}{2} + 669 = (n - 8)^2$$

On solving we get,

$$n^2 + 33n - 1210 = 0$$

(n-55)(n+22) = 0

n = 55

Number of balls = $\frac{n(n+1)}{2}$

$$= \frac{55(56)}{2} = 1540$$

Q19. Sum of all natural number in first 1000 integer which are not divisible by 2 are,

1+3+5+..... 999

n = 500

$S_{500} = \frac{500}{2}[1 + 999] = 250000$

Sum of all natural number in first 1000 integer which are not divisible by 5 are,

5+15+25+..... 995

n = 100

$S_{100} = \frac{100}{2}[5 + 995] = 50000$

Sum = 250000 - 50000 = 200000